VON MISES ELASTO-PLASTICITY

Fredy Andrés Mercado Navarro
Buenos Aires, Argentina

September 1, 2013

Abstract

Some personal notes on Von Mises Elasto-Plasticity theory.

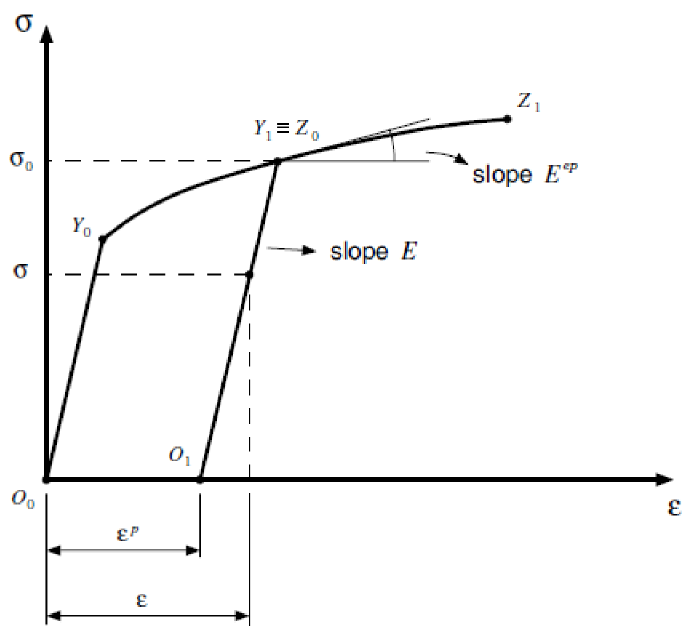

Figure 1: Elastoplastic Loading

1 INTRODUCTION

The problem to solve with plasticity is satisfying the yield condition at the end of each iteration. We must seek for the STRESS and PLASTIC STRAIN for time $t + \Delta t$.

Note that σ is any stress over the line O_1Y_1 (load-unload elastic curve). Also note that elastic module (Young) is the only one that exists in this theory to determine a stress state.

When the material is deformed plastically we re-define the yield stress point, but to reach that point we always follow the re-defined elastic curve. I always follow the constant slope line, which makes larger with plastic deformation, but always keeps the same slope E. We can say that:

$$\sigma = C^E \left(\epsilon - \epsilon^P\right)$$

The general counterpart of uniaxial elastic law (last equation) is given by:

$$\sigma = C^E : \epsilon^e = 2G\epsilon_d^e + K\epsilon_v^e \ I$$

Where C^E is the standard isotropic elasticity tensor and G and K are the shear and bulk moduli. The tensor ϵ_d^e is the deviatoric component of the elastic strain and $\epsilon_v^e = tr\left[\epsilon^e\right]$ is the volumetric elastic strain.

$${}_oS = C^E \left({}_o\epsilon - {}_o\epsilon^P\right)$$

2 VON MISSES PLASTICITY

In von Mises plasticity, the yield condition is at time $t + \Delta t$:

$${}^{t+\Delta t}f_y^{vM} = \frac{1}{2}{}^{t+\Delta t}S.{}^{t+\Delta t}S - \frac{1}{3}\left({}^{t+\Delta t}\sigma_y\right)^2 = 0$$

The material response is ELASTIC if:

$${}^t f_y < 0$$

And elastic or plastic (depends on load) if:

$${}^t f_y = 0$$

The above equation must hold throughout the PLASTIC response. When we are iterating in a computer we can have ${}^t f_y > 0$.

3 HARDENING LAW

The yield stress is a given function of the accumulated axial/equivalent plastic strain:

$$\sigma_y = \sigma_y\left(\bar{\epsilon}^p\right)$$
$${}^{t+\Delta t}\sigma_y = {}^t\sigma_y + \Delta\bar{\sigma}$$

$$^{t+\Delta t}\sigma_y = {}^{t}\sigma_y + E_p \Delta \bar{\epsilon}^p$$

$$^{t+\Delta t}\sigma_y = \frac{^{t}\sigma_y}{1 - \frac{2}{3} E_p \Delta \lambda}$$

Plastic flow at a generic yield limit produces a tangent relation between strain and stress. This relation is the Elastoplastic TENGENT modulus E^T. The Hardening slope or Hardening modulus is E_p (H in Figure).

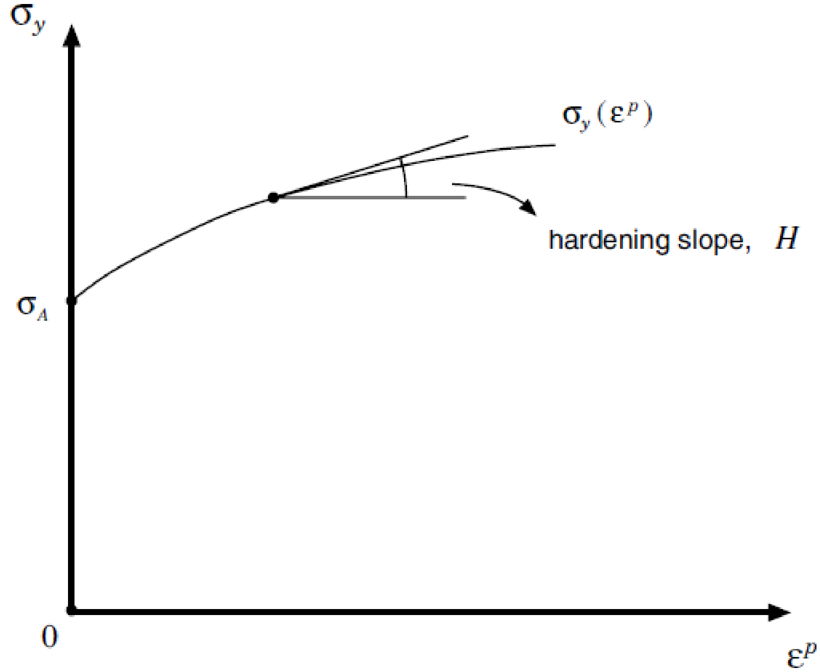

Figure 2: Hardening Curve (in general, not Bilinear)

3.1 STRAIN HARDENING MODULUS (TANGENT)

Is the tangent modulus E_T (see Figure). Hardening slope (see Hardening slope figure). For ADINA I need the Strain Hardening Modulus E_T, then:

$$E_T = \frac{E E_p}{E + E_p} = \frac{2.1e6x2100}{2.1e6 + 2100} = 2.0979e3 \left[\frac{kgf}{cm^2}\right]$$

In Lecture 17, online, Bathe says that E_T is the Strain Hardening Modulus. It is a slope, then:

$$Slope = \frac{y_2 - y_1}{x_2 - x_1}$$

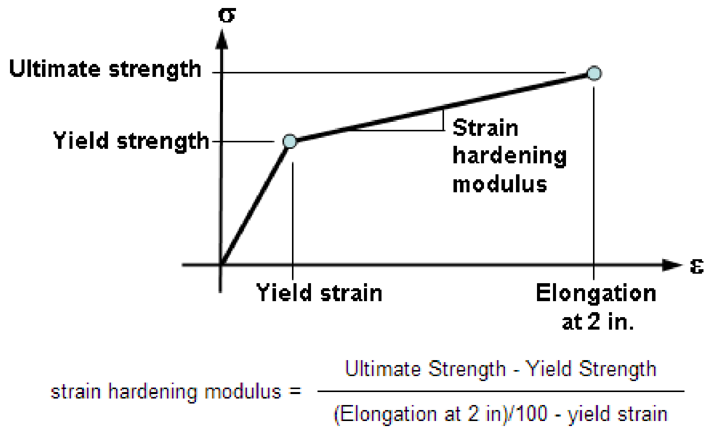

Figure 3: Strain Hardening Modulus

3.2 PERFECT PLASTICITY

$$E_T = E_P = 0$$

A material model is said to be perfectly plastic if no hardening is allowed, that is, the yield stress level does not depend in any way on the degree of plastification. In this case, the yield surface remains fixed regardless of any deformation process the material may experience and, in a uniaxial test, the elastoplastic modulus, E_T, vanishes. In the von Mises, Tresca, Drucker-Prager and Mohr-Coulomb models, perfect plasticity corresponds to a constant uniaxial yield stress, σ_y.

Perfectly plastic models are particularly suitable for the analysis of the stability of structures and soils and are widely employed in engineering practice for the determination of limit loads and safety factors.

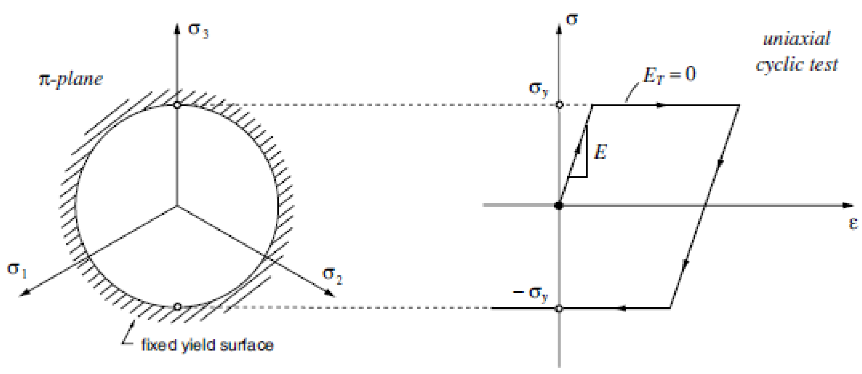

Figure 6.21. Perfect plasticity. Uniaxial test and π-plane representation.

Figure 4: Perfect Plasticity

3.3 KINEMATIC HARDENING

For a bilinear material, it means that when I unload and compress, I reach yield compressive point as $2\sigma_{ynew}$ starting on the tensile point.

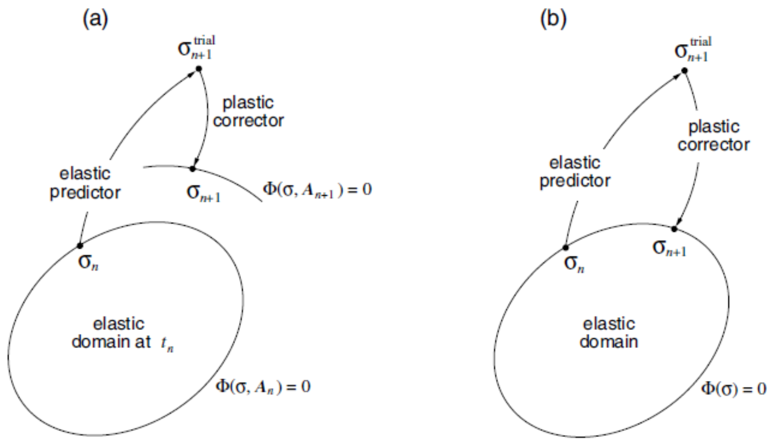

Figure 7.1. General return mapping schemes. Geometric interpretation: (a) hardening plasticity; and (b) perfect plasticity.

Figure 5: Return mapping for hardening plasticity and perfect plasticity

4 STRAINS

4.1 Plastic Strain

$$^{t+\Delta t}\underline{\epsilon}^p = {}^t\underline{\epsilon}^p + \Delta\underline{\epsilon}^p$$

4.2 Plastic Strain Increment

$$\Delta\underline{\epsilon}^p = \Delta\lambda^{t+\Delta t}\underline{s}$$

This allow us to calculate the elastic increment by substracting the elastic from the total deformation delta, which is given:

4.3 Equivalent Plastic Strain

$$^{t+\Delta t}\bar{\epsilon}^p = {}^t\bar{\epsilon}^p + \Delta\bar{\epsilon}^p$$

Remember that $^t\bar{\epsilon}^p = 0$ for the first step of the plastic corrector, where $\Delta\bar{\epsilon}^p$ is the equivalent plastic deformation increment.

I also have:

$$^t\bar{\epsilon}^P = \sqrt{\frac{2}{3}{}^t\underline{\epsilon}^P . {}^t\underline{\epsilon}^P}$$

4.4 Equivalent Plastic Strain Increment

$$\Delta\bar{\epsilon}^p = \sqrt{\frac{2}{3}(\Delta\lambda)^2 \left({}^{t+\Delta t}\underline{s}\right)^T \left({}^{t+\Delta t}\underline{s}\right)}$$

$$\Delta\bar{\epsilon}^p = \frac{2}{3}\Delta\lambda^{t+\Delta t}\sigma_y$$

4.5 Volumetric Strain

Taking the 3D constitutive elastic relation it can be demostrated, with:

$$p = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}$$

That I will get $p = \kappa\epsilon_{vol}$, where $\epsilon_{vol} = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$.

3D:

$$\Delta\epsilon_{vol} = \Delta\epsilon_{11} + \Delta\epsilon_{22} + \Delta\epsilon_{33}$$

Plain Strain:

$$\Delta\epsilon_{vol} = \Delta\epsilon_{11} + \Delta\epsilon_{22}$$

I have the math in my notes.

5 STRESSES

$${}^{t+\Delta t}\underline{\sigma} = {}^{t+\Delta t}\underline{s} + {}^{t+\Delta t}p\underline{m}$$

$${}^{t+\Delta t}p = {}^{t}p + \Delta p$$

$${}^{t}p = \frac{{}^{t}\sigma_{11} + {}^{t}\sigma_{22} + {}^{t}\sigma_{33}}{3}$$

$$\Delta p = \kappa\Delta\epsilon_{vol} = \kappa\Delta\underline{\epsilon}$$

$$\kappa = \frac{E}{3(1-2\nu)}$$

$$\Delta\epsilon_{vol} = \Delta\epsilon_{11} + \Delta\epsilon_{22} + \Delta\epsilon_{33}$$

$$\underline{m}^T = [\ 1\ 1\ 0\ 1\ ]$$

5.1 Equivalent Stress

$${}^{t}\bar{\sigma} = \sqrt{\frac{3}{2}\ {}^{t}\underline{\underline{s}} : {}^{t}\underline{\underline{s}}}$$

$${}^{t}\underline{\underline{s}} : {}^{t}\underline{\underline{s}} = \left({}^{t}s_{11}\right)^2 + \left({}^{t}s_{22}\right)^2 + \left({}^{t}s_{33}\right)^2 + 2\left({}^{t}s_{12}\right)^2$$

5.2 Equivalent Stress Increment

$$\Delta\bar{\sigma} = E_p \Delta\bar{\epsilon}^p$$

5.3 Stress Increment during plastic yield

Can be calculated with $\Delta\epsilon$. Also with $\Delta\epsilon^p$. With E_T curve we can do:

$$\Delta\bar{\sigma}^p = E_T \Delta\bar{\epsilon} = E_p \Delta\bar{\epsilon}^p$$

We can make the equivalence:

$$\Delta\underline{\sigma}^p = E_T \Delta\underline{\epsilon} = E_p \Delta\underline{\epsilon}^p$$

It is a plastic delta sigma because its slope is E_T or E_p, so this delta is over the plastic curve. It means that there exists a stress increment and that it is a function of the plastic deformation.

5.4 Hydrostatic Stress (Pressure)

The return mapping affects only the deviatoric stress component. The hydrostatic stress, p_{n+1}, has the value computed in the elastic predictor stage and can, therefore, be eliminated from the system of equations.

Está así en el algoritmo de Owen.

$${}^t p = \frac{{}^t\sigma_{11} + {}^t\sigma_{22} + {}^t\sigma_{33}}{3}$$

6 DEVIATORIC STRESS

$$\sigma_{ij} = s_{ij} + \frac{\sigma_{kk}}{3}\delta_{ij}$$

Then:

$$s_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij} = \sigma_{ij} - p\delta_{ij}$$
$$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$$
$$p = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}$$

6.1 For 3D

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} - \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} = \begin{bmatrix} \sigma_{11} - p & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - p & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - p \end{bmatrix}$$

6.2 For 2D

For Plain Strain:

$$\begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & 0 \\ 0 & 0 & s_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} - \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} = \begin{bmatrix} \sigma_{11} - p & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} - p & 0 \\ 0 & 0 & \sigma_{33} - p \end{bmatrix}$$

$$s_{11} = \frac{2}{3}\sigma_{11} - \left(\frac{\sigma_{22} + \sigma_{33}}{3}\right)$$

$$s_{22} = \frac{2}{3}\sigma_{22} - \left(\frac{\sigma_{11} + \sigma_{33}}{3}\right)$$

$$s_{12} = \sigma_{12}$$

$$s_{33} = \frac{2}{3}\sigma_{33} - \left(\frac{\sigma_{11} + \sigma_{22}}{3}\right)$$

For Plain Stress ($\sigma_{33} = 0$):

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} - \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} = \begin{bmatrix} \sigma_{11} - p & \sigma_{12} \\ \sigma_{21} & \sigma_{22} - p \end{bmatrix}$$

Pressure results from dividing by 3:

$$p = \frac{\sigma_{11} + \sigma_{22}}{3}$$

6.3 For the algoritm

$$^{t+\Delta t}\underline{s} = \frac{\underline{s}^{trial}}{(1 + 2G\Delta\lambda)}$$

7 RADIAL RETURN ALGORITHM (MAPPING)

First of all, there are three situations that can occur. First, strain increment can be all ELASTIC, second, strain increment can be ELASTIC and PLASTIC, and third, strain increment can be all PLASTIC.

Procedure used to calculate the total stresses at time $t + \Delta t$.

Given:
$^{t}\underline{\epsilon}$: Total strains at time t.
$^{t+\Delta t}\underline{\epsilon}$: Total strains at time $t + \Delta t$.
$^{t}\underline{\sigma}$: Total stresses at time t.
$^{t}\sigma_y$: Yield stress at time t.

(a) Calculate the strain increment.

$$\Delta\underline{\epsilon} = {}^{t+\Delta t}\underline{\epsilon} - {}^{t}\underline{\epsilon}$$

(b) Calculate the stress increment $\Delta\underline{\sigma}$, assuming elastic behavior:

$$\Delta\underline{\sigma} = C^E \Delta\underline{\epsilon}$$

I know that, in general, for Plain Strain $\epsilon_{33} = 0$, so $\Delta\epsilon_{33} = 0$, and:

$$\begin{bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{12} \end{bmatrix} = \left[C^E\right] \begin{bmatrix} \Delta\epsilon_{11} \\ \Delta\epsilon_{22} \\ 2\Delta\epsilon_{12} \end{bmatrix}$$

We have a Plain Strain case, but we can calculate $\Delta\sigma_{33}$:

$$\Delta\sigma_{33} = \frac{E\nu}{(1+\nu)(1-2\nu)}(\Delta\epsilon_{11} + \Delta\epsilon_{22})$$

So the complete trial state of stress is:

$$\Delta\underline{\sigma}^e = \begin{bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{12} \\ \Delta\sigma_{33} \end{bmatrix}$$

7.1 Elastic Predictor

(c) Calculate σ^{trial}, assuming ELASTIC behavior:

$$\sigma^{trial} = {}^{t}\underline{\sigma} + \Delta\underline{\sigma}^e$$

Here $\Delta\epsilon^P$ is assumed to be zero.

(d) With σ^{trial} as the state of stress, calculate the value of the yield function f^{trial}.

$$f^{trial} = \frac{1}{2}\left(\underline{s}^{trial}\right)^T \left(\underline{s}^{trial}\right) - \frac{\left({}^{t}\sigma_y\right)^2}{3}$$

$$\underline{s}^{trial} = \begin{bmatrix} s_{11}^{trial} \\ s_{22}^{trial} \\ s_{12}^{trial} \\ s_{33}^{trial} \end{bmatrix}$$

$$p^{trial} = \frac{(\sigma_{11}^{trial} + \sigma_{22}^{trial} + \sigma_{33}^{trial})}{3}$$

Donde:

$$s_{11}^{trial} = \frac{2}{3}\sigma_{11}^{trial} - \left(\frac{\sigma_{22}^{trial} + \sigma_{33}^{trial}}{3}\right) = \sigma_{11}^{trial} - p^{trial}$$

$$s_{22} = \frac{2}{3}\sigma_{22}^{trial} - \left(\frac{\sigma_{11}^{trial} + \sigma_{33}^{trial}}{3}\right) = \sigma_{22}^{trial} - p^{trial}$$

$$s_{12}^{trial} = \sigma_{12}^{trial}$$

$$s_{33}^{trial} = \frac{2}{3}\sigma_{33}^{trial} - \left(\frac{\sigma_{11}^{trial} + \sigma_{22}^{trial}}{3}\right) = \sigma_{33}^{trial} - p^{trial}$$

(e) If $f^{trial} <= 0$, the strain increment is ELASTIC, so $\Delta\underline{\epsilon} = \Delta\underline{\epsilon}^e$. In this case, σ^{trial} is correct; we return.

$$^{t+\Delta t}\underline{\epsilon}^e = {}^t\underline{\epsilon}^e + \Delta\underline{\epsilon}^e = {}^t\underline{\epsilon}^e + \Delta\underline{\epsilon}$$

$$^{t+\Delta t}\underline{\epsilon}^p = {}^t\underline{\epsilon}^p + \Delta\underline{\epsilon}^p = {}^t\underline{\epsilon}^p + 0 = {}^t\underline{\epsilon}^p$$

$$^{t+\Delta t}\underline{\epsilon} = {}^{t+\Delta t}\underline{\epsilon}^e + {}^{t+\Delta t}\underline{\epsilon}^p$$

$$^{t+\Delta t}\underline{\sigma} = {}^t\underline{\sigma} + \Delta\underline{\sigma}^e = \underline{\sigma}^{trial}$$

$$^{t+\Delta t}\bar{\epsilon}^p = {}^t\bar{\epsilon}^p + \Delta\bar{\epsilon}^p = {}^t\bar{\epsilon}^p + 0 = {}^t\bar{\epsilon}^p$$

$$^{t+\Delta t}\sigma_y = {}^t\sigma_y$$

Because there was no $\Delta\underline{\epsilon}^p$, and I have to add the new all elastic $\Delta\underline{\epsilon}$ to the accumulated elastic strain.

7.2 Plastic Corrector

$$\Delta\lambda = \frac{\left[\frac{3}{2}\left(\underline{s}^{trial}\right)^T\underline{s}^{trial}\right]^{\frac{1}{2}} - {}^t\sigma_y}{2G^t\sigma_y + \frac{2}{3}\left[\frac{3}{2}\left(\underline{s}^{trial}\right)^T\underline{s}^{trial}\right]^{\frac{1}{2}} E_p}$$

Owen, p. 236:

$$\left[\frac{3}{2}\left(\underline{s}^{trial}\right)^T\underline{s}^{trial}\right]^{\frac{1}{2}} = \bar{s}^{trial} = \left[s_{11}s_{11} + s_{22}s_{22} + 2s_{12}s_{12} + s_{33}s_{33}\right]^{\frac{1}{2}}$$

$$^{t+\Delta t}\underline{s} = \frac{\underline{s}^{trial}}{(1 + 2G\Delta\lambda)}$$

Stresses:

$$^{t+\Delta t}\underline{\sigma} = {}^{t+\Delta t}\underline{s} + {}^{t+\Delta t}p\underline{m}$$

$$\underline{m}^T = [\ 1\ 1\ 0\ 1\ ]$$

$$^{t+\Delta t}p = {}^{t}p + \Delta p = p^{trial}$$

Owen, pag. 221. p^{trial}. Radial return algorithm only affects deviatoric components.

$$^{t}p = \frac{^{t}\sigma_{11} + {}^{t}\sigma_{22} + {}^{t}\sigma_{33}}{3}$$
$$\Delta p = \kappa \Delta \epsilon_{vol}$$
$$\kappa = \frac{E}{3(1-2\nu)}$$

Remember that for 3D $\Delta \underline{\epsilon}$ it is given:

$$\Delta \epsilon_{vol} = \Delta \epsilon_{11} + \Delta \epsilon_{22} + \Delta \epsilon_{33}$$

For Plain Strain:

$$\Delta \epsilon_{vol} = \Delta \epsilon_{11} + \Delta \epsilon_{22}$$

With teacher notes we find $\Delta \underline{\epsilon}^{p}$, while with RATIO of Bathe we find $\Delta \underline{\epsilon}^{e}$, because RATIO is a number that gives the elastic fraction of deformation delta.

UPDATES

$$\Delta \underline{\epsilon}^{p} = (\Delta \epsilon - \Delta \epsilon^{e}) = \Delta \lambda^{t+\Delta t}\underline{s}$$

Before I just took 11, 22 and 12 componentes of deviatoric stress.

$$^{t+\Delta t}\underline{\epsilon}^{e} = {}^{t}\underline{\epsilon}^{e} + \Delta \underline{\epsilon}^{e} = {}^{t}\underline{\epsilon}^{e} + (\Delta \underline{\epsilon} - \Delta \underline{\epsilon}^{p})$$
$$^{t+\Delta t}\underline{\epsilon}^{p} = {}^{t}\underline{\epsilon}^{p} + \Delta \underline{\epsilon}^{p} = {}^{t}\underline{\epsilon}^{p} + (\Delta \epsilon - \Delta \epsilon^{e}) = {}^{t}\underline{\epsilon}^{p} + \Delta \lambda^{t+\Delta t}\underline{s}$$
$$^{t+\Delta t}\underline{\epsilon} = {}^{t+\Delta t}\underline{\epsilon}^{e} + {}^{t+\Delta t}\underline{\epsilon}^{p}$$

$$^{t+\Delta t}\bar{\epsilon}^{p} = {}^{t}\bar{\epsilon}^{p} + \Delta \bar{\epsilon}^{p} = {}^{t}\bar{\epsilon}^{p} + \frac{2}{3}\Delta \lambda^{t+\Delta t}\sigma_{y} = {}^{t}\bar{\epsilon}^{p} + \sqrt{\frac{2}{3}(\Delta \lambda)^{2}\left(^{t+\Delta t}\underline{s}\right)^{T}\left(^{t+\Delta t}\underline{s}\right)} = {}^{t}\bar{\epsilon}^{p} + \sqrt{\frac{2}{3}\Delta \underline{\epsilon}^{p}.\Delta \underline{\epsilon}^{p}}$$

I find some inconsistencies here. tdtS has 4 components and deltaEPS three.

$$^{t+\Delta t}\underline{\sigma} = {}^{t}\underline{\sigma} + \Delta \underline{\sigma}^{e} = {}^{t}\underline{\sigma} + C^{E}\Delta \underline{\epsilon}^{e} = {}^{t}\underline{\sigma} + C^{E}\left(\Delta \underline{\epsilon} - \Delta \underline{\epsilon}^{p}\right) = \sigma^{trial} - C^{E}\Delta \underline{\epsilon}^{p}$$
$$^{t+\Delta t}\sigma_{y} = {}^{t}\sigma_{y} + \Delta \bar{\sigma}^{p} = {}^{t}\sigma_{y} + E_{p}\Delta \bar{\epsilon}^{p} = \frac{^{t}\sigma_{y}}{1 - \frac{2}{3}E_{p}\Delta \lambda} = {}^{t+\Delta t}\bar{\sigma} = \sqrt{\frac{3}{2}{}^{t+\Delta t}\underline{s}.{}^{t+\Delta t}\underline{s}}$$

Equation $^{t+\Delta t}\underline{\sigma}$ is important. See figure RR1. If I substract tension from elastic deformation delta I get $^{t+\Delta t}\sigma$ right over the yield line.

8 SUMMARY OF STRESS CALCULATION

Procedure used to calculate the total stresses at time $t + \Delta t$.

Given:
$STRAIN$: Total strains at time $t + \Delta t$: ${}^{t+\Delta t}\underline{\epsilon}$.
SIG: Total stresses at time t: ${}^{t}\underline{\sigma}$.
EPS: Total strains at time t: ${}^{t}\underline{\epsilon}$

(a) Calculate the strain increment.

$$DELEPS = STRAIN - EPS$$

$$\Delta\underline{\epsilon} = {}^{t+\Delta t}\underline{\epsilon} - {}^{t}\underline{\epsilon}$$

(b) Calculate the stress increment $DELSIG$, assuming elastic behavior:

$$DELSIG = C^E * DELEPS$$

$$\Delta\underline{\sigma} = C^E \Delta\underline{\epsilon}$$

(c) Calculate TAU, assuming elastic behavior:

$$TAU = SIG + DELSIG$$

$$\sigma^{trial} = {}^{t}\underline{\sigma} + \Delta\underline{\sigma}$$

(d) With $TAU\left(\sigma^{trial}\right)$ as the state of stress, calculate the value of the yield function F.
(e) If $F(TAU) <= 0$, the strain increment is elastic. In this case, TAU is correct; we return.

(f) If $F(TAU) > 0$. If the previous state of stress was plastic, set $RATIO$ to zero and go to (g), because the whole strain was taken up plastically.

Otherwise, if it is the FIRST TIME that the yielding occurs (we have to find that stress value at which yielding started, and we do so by entering the yield function with $F(SIG + RATIO * DELSIG) = 0$ or $F({}^{t}\underline{\sigma} + RATIO * \Delta\underline{\sigma}) = 0$ where $0 < RATIO <= 1$ is unknown, and is a number that is between 0 and 1 because $\Delta\underline{\sigma}$ produced yielding, so there is a part of $\Delta\underline{\sigma}$ that causes ELASTIC deformation and a part that causes PLASTIC deformation. SIG is known, $DELSIG$ is known, so we can calculate from yield equation $RATIO$ and find the stress increment that brings us to the initiation of yielding), there is a transition from elastic to plastic and $RATIO$ (the portion of incremental strain taken elastically) has to be determined. $RATIO$ is determined from:

$$F(SIG + RATIO * DELSIG) = 0$$

$$F({}^{t}\underline{\sigma} + RATIO * \Delta\underline{\sigma}) = 0$$

Since $F = 0$ signals the initiation of yielding.

(g) Redefine $TAU(\sigma^{trial})$ as the stress at start of yield:

$$TAU = SIG + RATIO * DELSIG$$
$$\sigma^{trial} = {}^{t}\sigma + RATIO * \Delta\sigma$$

and calculate the elastic-plastic strain increment (DEPS=total elastic-plastic strain increment):

$$DEPS = (1 - RATIO) * DELEPS$$
$$\Delta\epsilon^{p} = (1 - RATIO) * \Delta\epsilon$$
$$\Delta\epsilon^{e} = RATIO * \Delta\epsilon$$
$$\Delta\epsilon^{e} + \Delta\epsilon^{p} = \Delta\epsilon$$

(h) Divide DEPS into subincrements DDEPS and calculate (the stress occuring during the plastic deformation TAU):

$$TAU = TAU + C^{EP} * DDEPS$$

for all elastic-pastic strain subincrements.

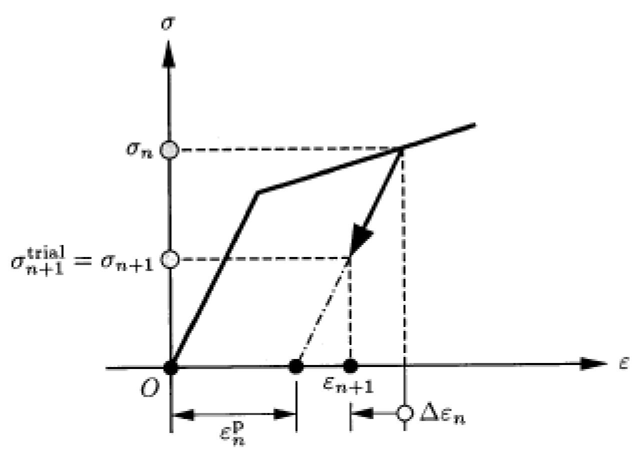

Figure 6: RR1

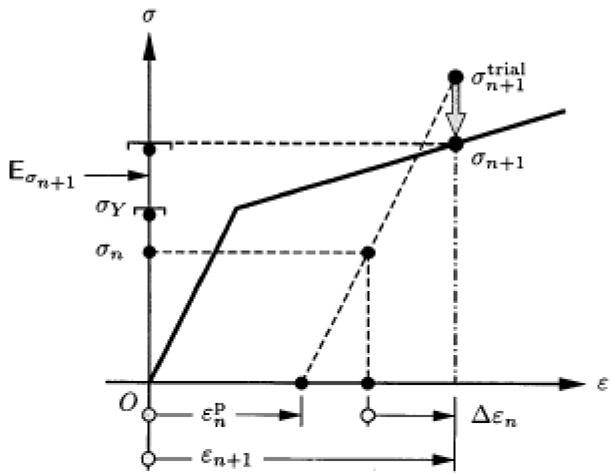

FIGURE 1-11. The trial state violates the constraint condition $f \leq 0$. Consequently, the incremental process must be plastic since $\Delta\gamma > 0$ to achieve $\sigma_{n+1} \neq \sigma_{n+1}^{\text{trial}}$.

Figure 7: RR2

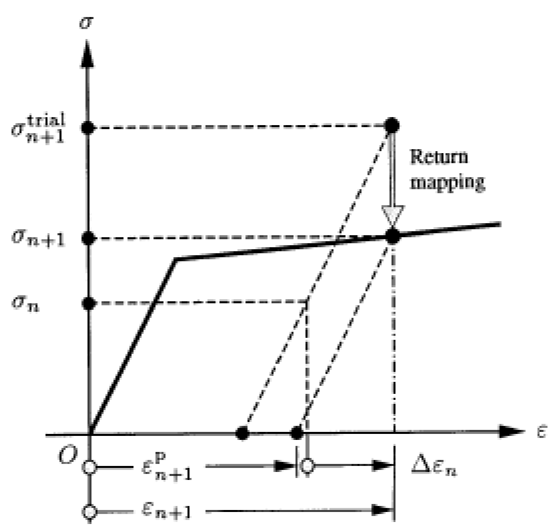

FIGURE 1-12. The final stress is obtained by "*returning*" the trial stress to the yield surface through a scaling, hence, the denomination return mapping.

Figure 8: RR3

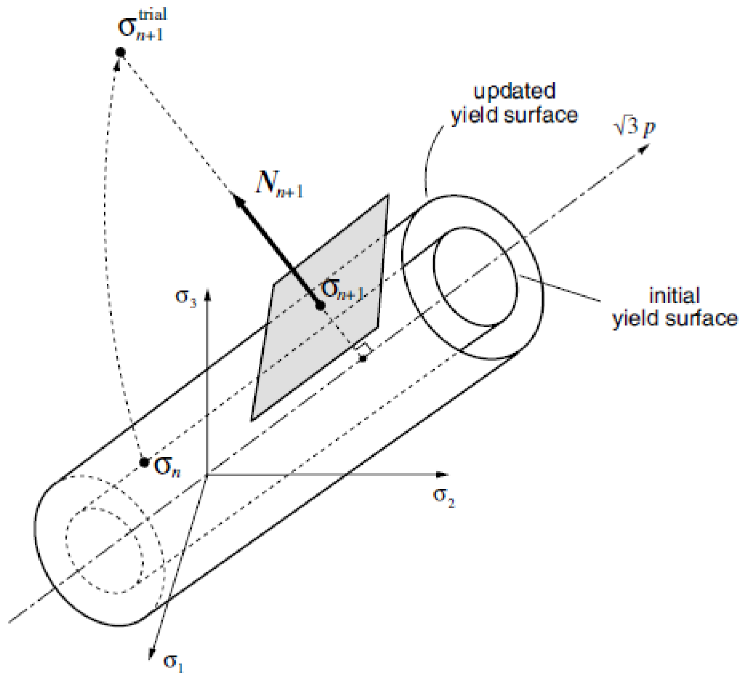

Figure 7.8. The implicit elastic predictor/return-mapping scheme for the von Mises model. Geometric interpretation in principal stress space.

Figure 9: RR4

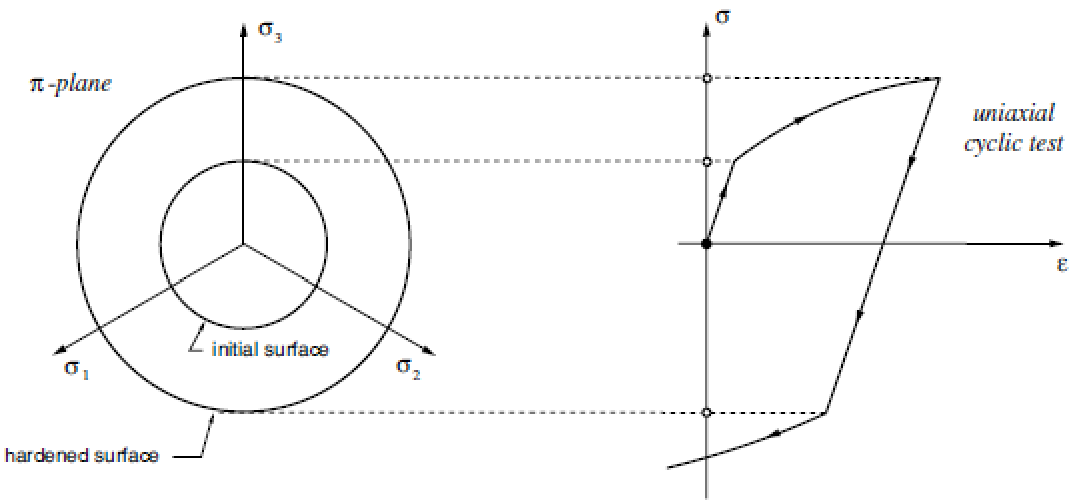

Figure 6.22. Isotropic hardening. Uniaxial test and π-plane representation.

Figure 10: Isotropic Hardening