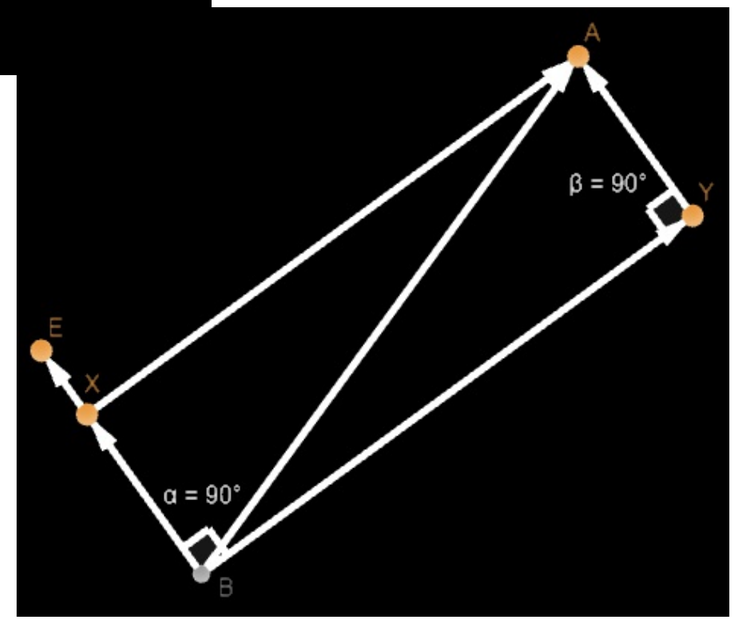
Sean $\vec{a} = (-1,5,7)$ y $\vec{e} = (1,-2,3)$. Descomponga $\vec{a}$ en la suma de dos vectores libres $\vec{x}$ y $\vec{y}$, tales que $\vec{x}$ sea paralelo a $\vec{e}$ y $\vec{y}$ sea ortogonal a $\vec{e}$. Hallar los vectores $\vec{x}$ y $\vec{y}$.



Cond. 1. $\vec{a} = \vec{x} + \vec{y}$

Cond. 2. $\vec{x} \parallel \vec{e}$.

Cond. 3. $\vec{y} \perp \vec{e}$.

C1. $\begin{bmatrix} -1 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \rightarrow$

$-1 = x_1 + y_1$ (1)

$5 = x_2 + y_2$. (2)

$7 = x_3 + y_3$. (3)

C2. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = K \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \rightarrow$

$x_1 = K$ (4)

$x_2 = -2K$. (5)

$x_3 = 3K$. (6)

C3. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0 \rightarrow y_1(1) + y_2(-2) + y_3(3) = 0$

$y_1 - 2y_2 + 3y_3 = 0$. (7)

④, ⑤ y ⑥ en ①, ② y ③:

$-1 = K + Y_1 \rightarrow Y_1 = -1-K.$ ⑧

$5 = -2K + Y_2 \rightarrow Y_2 = 5+2K.$ ⑨

$7 = 3K + Y_3. \rightarrow Y_3 = 7-3K.$ ⑩

⑧, ⑨ y ⑩ en ⑦:

$(-1-K) - 2(5+2K) + 3(7-3K) = 0$

$-1 - K - 10 - 4K + 21 - 9K = 0$

$10 - 14K = 0$

$\frac{10}{14} = K = \frac{5}{7}$ ⑪

⑪ en ⑧, ⑨ y ⑩:

⑫ $Y_1 = -1 - \frac{5}{7} = -\frac{7}{7} - \frac{5}{7} = -\frac{12}{7}.$

⑬ $Y_2 = 5 + 2 \cdot \frac{5}{7} = 5 + \frac{10}{7} = \frac{35}{7} + \frac{10}{7}$

$Y_2 = \frac{45}{7}.$

⑭ $Y_3 = 7 - 3 \cdot \frac{5}{7} = \frac{49}{7} - \frac{15}{7} = \frac{34}{7}$

⑫, ⑬ y ⑭ en ①, ② y ③:

$X_1 = -1 - \left(-\frac{12}{7}\right) = -\frac{7}{7} + \frac{12}{7} = \frac{5}{7}$

$X_2 = 5 - \frac{45}{7} = \frac{35}{7} - \frac{45}{7} = -\frac{10}{7}$

$X_3 = 7 - \frac{34}{7} = \frac{49}{7} - \frac{34}{7} = \frac{15}{7}$

$$\vec{X} = \begin{bmatrix} \frac{5}{7} \\ -\frac{10}{7} \\ \frac{15}{7} \end{bmatrix}$$

$$\vec{Y} = \begin{bmatrix} -\frac{12}{7} \\ \frac{45}{7} \\ \frac{34}{7} \end{bmatrix}$$