CONTINUUM MECHANICS NOTES

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September 1, 2013

Abstract

General and Personal Notes on Continuum Mechanics for Finite Element Analysis.

1 DEFORMATION GRADIENT TENSOR

$$ {}_0^tX = \begin{bmatrix} \frac{\partial^t x_1}{\partial^0 x_1} & \frac{\partial^t x_1}{\partial^0 x_2} \\ \frac{\partial^t x_2}{\partial^0 x_1} & \frac{\partial^t x_2}{\partial^0 x_2} \end{bmatrix}. $$

An alterntive form is:

$$ {}_0^tX_{\alpha\beta} = \delta_{\alpha\beta} + \frac{\partial^t u^\alpha}{\partial^o z^\beta} $$

$$ {}_o^tX = I + \begin{bmatrix} \frac{\partial^t u_1}{\partial^o x_1} & \frac{\partial^t u_1}{\partial^o x_2} \\ \frac{\partial^t u_2}{\partial^o x_1} & \frac{\partial^t u_2}{\partial^o x_2} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\partial^t u_1}{\partial^o x_1} & \frac{\partial^t u_1}{\partial^o x_2} \\ \frac{\partial^t u_2}{\partial^o x_1} & 1 + \frac{\partial^t u_2}{\partial^o x_2} \end{bmatrix} $$

$$ \begin{bmatrix} \frac{\partial^t u_i}{\partial^o x_1} \\ \frac{\partial^t u_i}{\partial^o x_2} \end{bmatrix} = \frac{1}{det(^oJ)} \begin{bmatrix} \frac{\partial^o x_2}{\partial s} & -\frac{\partial^o x_2}{\partial r} \\ -\frac{\partial^o x_1}{\partial s} & \frac{\partial^o x_1}{\partial r} \end{bmatrix} \begin{bmatrix} \frac{\partial^t u_i}{\partial r} \\ \frac{\partial^t u_i}{\partial s} \end{bmatrix} $$

$$ det(^oJ) = \frac{\partial^o x_1}{\partial r}\frac{\partial^o x_2}{\partial s} - \frac{\partial^o x_1}{\partial s}\frac{\partial^o x_2}{\partial r} $$

2 SECOND PIOLA-KIRCCHOFF STRESS TENSOR

$$ {}_0^tS = \frac{^0\rho}{^t\rho}\, {}_t^0X\ {}^t\tau\ {}_t^0X^T $$

$$ {}_0^tS_{ij} = \frac{^0\rho}{^t\rho}\, {}_t^0X_{i,m}\ {}^t\tau_{mn}\ {}_t^0X^T_{j,n} $$

This is a stress that has been "pulled back" to the reference configuration and referred to area elements there, so we have to do a "push forward" to obtain Cauchy stresses.

"The second Piola-Kirchhoff stress and Green-Lagrange strain components do not change measured in a fixed coordinate system when the material is subjected to rigid body MOTIONS".

3 GREEN-LAGRANGE STRAIN TENSOR

If the nonlinear portion of the expression is neglected, one obtains the infinitesimal strains encountered in linear finite element analysis.

In cartesian coordinates ${}^{t}_{0}\epsilon$ can be calculated as:

$$\left[{}^{t}_{o}\epsilon\right] = \begin{bmatrix} {}^{t}_{o}\epsilon_{11} & {}^{t}_{o}\epsilon_{12} & {}^{t}_{o}\epsilon_{13} \\ {}^{t}_{o}\epsilon_{21} & {}^{t}_{o}\epsilon_{22} & {}^{t}_{o}\epsilon_{23} \\ {}^{t}_{o}\epsilon_{31} & {}^{t}_{o}\epsilon_{32} & {}^{t}_{o}\epsilon_{33} \end{bmatrix} = \frac{1}{2}\left[{}^{t}_{o}X^{T}\ {}^{t}_{o}X - I\right] = \frac{1}{2}\left[{}^{t}_{o}X^{T}\ {}^{t}_{o}X - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

Como vector:

$$\left[{}^{t}_{o}\epsilon\right] = \begin{bmatrix} {}^{t}_{o}\epsilon_{11} \\ {}^{t}_{o}\epsilon_{22} \\ {}^{t}_{o}\epsilon_{33} \\ 2{}^{t}_{o}\epsilon_{12} \\ 2{}^{t}_{o}\epsilon_{23} \\ 2{}^{t}_{o}\epsilon_{13} \end{bmatrix}$$